**Quiz (2) First year – Electrical Engineering (power)**

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## Answer the following question:

**(1)** If $\vec{A}=\vec{\nabla}\phi$ where $\phi=z^2x^3y$ find $\vec{\nabla}\times\vec{A},\ \vec{\nabla}.\vec{A}$.

**(2)** Evaluate $\int_C \vec{F}.d\vec{r}$ from $(1,0,1)$ to $(0,1,1)$ along $x^2+y^2=1,\ z=1$ for

$\vec{F}=(yz+2x)\vec{i}+(xz)\vec{j}+(xy+2z)\vec{k}$.

**(3)** Evaluate $\iint_S \vec{F}.\vec{n}dS$ where $\vec{F}=2y\ \vec{i}-z\ \vec{j}+x^2\vec{k}$ and $S$ determined by

$y^2=8x,\quad x,y,z\geq 0,\ y=4, z=6$

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**Answer (1)**

$$\bar{A}=\bar{\nabla}\varphi=\frac{\partial\varphi}{\partial x}\bar{i}+\frac{\partial\varphi}{\partial y}\bar{j}+\frac{\partial\varphi}{\partial z}\bar{k}$$

$$=\frac{\partial}{\partial x}\left(z^2x^3y\right)\bar{i}+\frac{\partial}{\partial y}\left(z^2x^3y\right)\bar{j}+\frac{\partial}{\partial z}\left(z^2x^3y\right)\bar{k}=3z^2x^2y\,\vec{i}+z^2x^3\bar{j}+2z\,x^3y\,\vec{k}$$

$$\vec{\nabla}\times\vec{A}=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ 3z^2x^2y & z^2x^3 & 2z\,x^3y\end{vmatrix}=0$$

$$\vec{\nabla}.\vec{A}=\left(\frac{\partial}{\partial x}\bar{i}+\frac{\partial}{\partial y}\bar{j}+\frac{\partial}{\partial z}\bar{k}\right).\left(3z^2x^2y\,\vec{i}+z^2x^3\bar{j}+2z\,x^3y\,\vec{k}\right)=6z^2xy+2x^3y$$

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**Answer: (2)**

Evaluate $\int_C \vec{F}.d\vec{r}$ from $(1,0,1)$ to $(0,1,1)$ along $x^2+y^2=1,\ z=1$ for

$\vec{F}=(yz+2x)\vec{i}+(xz)\vec{j}+(xy+2z)\vec{k}$.

**Answer:**

$$\int_C \vec{F}.d\vec{r}=\int_C (yz+2x)dx+(xz)dy+(xy+2z)dz$$

Since $z=1$ then $dz=0$ the integral becomes

$$\int_C (y+2x)dx+(x)dy \text{ along the circle } x^2+y^2=1$$

$$\int_C (y+2x)dx+(x)dy = \int_{(1,0)}^{(0,1)} (y+2x)dx+(x)dy$$

$$= \int_0^{\pi/2} \left[(\sin\theta+2\cos\theta)(-\sin\theta d\theta)+\cos\theta\cos\theta\right]d\theta = \int_0^{\pi/2} \left[-(\sin^2\theta+2\cos\theta\sin\theta)+\cos^2\theta\right]d\theta$$

$$= \int_0^{\pi/2} (\cos 2\theta-\sin 2\theta)d\theta = \frac{1}{2}\left[\sin 2\theta+\cos 2\theta\right]_0^{\pi/2} = \frac{1}{2}\left[-1-1\right] = -1$$

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**Answer (3)**

Evaluate $\iint_S \vec{F}.\vec{n}dS$ where $\vec{F} = 2y\,\vec{i} - z\,\vec{j} + x^2\vec{k}$ and $S$ determined by

$y^2 = 8x,\ \ x,y,z \ge 0,\ y = 4, z = 6$.

**Answer:**

$$\varphi = y^2 - 8x, \qquad \overline{\nabla}\varphi = -8\overline{i} + 2y\,\overline{j}, \qquad \overline{n} = \frac{-8\overline{i} + 2y\,\overline{j}}{\sqrt{68}}, \quad \overline{n}.\overline{i} = \frac{-8}{\sqrt{68}}$$

$$\overline{F}.\overline{n} = \left(2y\,\vec{i} - z\,\vec{j} + x^2\vec{k}\right).\left(\frac{-8\overline{i} + 2y\,\overline{j}}{\sqrt{68}}\right) = \frac{-16y - 2yz}{\sqrt{68}}$$

$$dS = \frac{dzdy}{\overline{n}.\overline{i}} = \frac{\sqrt{68}}{-8}\,dzdy$$

$$\iint_S \vec{F}.\vec{n}dS = \iint_A \left(\frac{-16y - 2yz}{\sqrt{68}}\right)\frac{\sqrt{68}}{-8}\,dzdy = \iint_A \left(\frac{-16y - 2yz}{-8}\right)dzdy$$

Where A is the rectangle with vertices (0,0,0),(0,4,0), (0,4,6) and (0,0,6) in zy- plane

$$\iint_A \left(\frac{-16y - 2yz}{-8}\right)dzdy = \frac{-1}{8}\int_{y=0}^{y=4}\int_{z=0}^{6} y\left(-16-2z\right)dxdy$$

$$= \frac{-1}{8}\left[\frac{1}{2}y^2\right]_0^4\left[\frac{\left(-16-2z\right)^2}{-4}\right]_0^6 = \frac{1}{64}[16]\left[\left(-28\right)^2 - (-16)^2\right] = 132$$